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TRANSPORT PROCESSES IN A REACTING BOUNDARY LAYER

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An approximate calculation has been made for the boundary layer of a compressible gas in a laminar flow over a semi-infinite porous plate reacting with an injected substance in a homogeneous reaction at an infinite rate. The possibility of applying the solutions obtained to a turbulent boundary layer is also examined.

Heat and mass transfer processes in a boundary layer in the presence of chemical reactions are of great practical interest. There are many examples of apparatus and equipment where use is made of subliming surfaces, and also systems with gas injected through a porous wall, in which the sublimed or injected substances may reach the boundary layer and react with it.

In the analytical research [1, 2, 5, 9, 10] dealing with this matter, attention has been given in the main to flow over a body, through the porous walls of which coolant is injected according to the law $v_w \approx x^{1/2}$ or $v_w \approx x^{-1/2}$ to react in the laminar boundary layer with the incoming stream. A similar problem is examined in [8] for the case of uniform injection v_w = const of a foreign substance along the length of a porous plate. There it is assumed that the plate is immersed in an incompressible gas with $\mathrm{Pr} = \mathrm{Pr}_m = 1$.

In the present paper an approximate analytical investigation is made of the heat and mass transfer processes in a laminar boundary layer into which a substance is uniformly v_w = const injected through a porous semi-infinite plate to react with the steady high-speed gas flow. The solution is given for the case of an infinite reaction rate v_r at a certain temperature T_* on some surface $y = y_*(x)$, which is a surface of discontinuity in the boundary layer.

It is known that if $v_r \gg v_d$, where v_d is the diffusion rate, then the reaction occurs in a well-defined zone (reaction front) [1, 2] within the reacting boundary layer.

The thermal and diffusion Prandtl numbers (Pr, Pr_m) are assumed to be variable or constant and not equal to unity, i.e., $\mathrm{Pr} \neq 1$ = const, $\mathrm{Pr}_m \neq 1$ = const, $\mathrm{Pr} \neq \mathrm{Pr}_m$. It is assumed that the effective physical coefficients of the gas mixture are uniform throughout the boundary layer, that the mean molecular weight of the mixture does not differ much from the molecular weight of the components, and that the rate of arrival of oxidizer (usually oxygen) from the external flow is somewhat greater than the stoichiometric value, as a result of which the progress of the reaction depends only on diffusion of the injected gas.

Under the above assumptions (thermal diffusion not taken into account), the equations of the laminar boundary layer for reacting gas mixtures may be written in the form

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0, \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right), \tag{2}$$

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left(\frac{\mu}{\mathrm{Pr}} \frac{\partial h}{\partial y}\right), \tag{3}$$

$$\rho u \frac{\partial C_1}{\partial x} + \rho v \frac{\partial C_1}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu}{\mathrm{Pr}_m} \frac{\partial C_1}{\partial y}\right), \tag{4}$$

$$C_1 + C_2 + C_3 = 1, \tag{5}$$

where $C_1 = \rho_1/\rho$, C_2, C_3 are, respectively, the concentrations by weight of the fuel gas, the reaction products, and the inert component; h is the enthalpy $h = c_p T$.

We assume a linear dependence of viscosity on temperature $\mu = \mu_0 T/T_0$, where μ_0 and T_0 are constants, the diffusion coefficient $D = D(T)$, the thermal conductivity $\lambda = \lambda(T)$, and c_p = const.

We transform (1)–(4) to the Crocco variable [3] $u = u(x, y)$ $x = x$. After elimination of v, Eqs. (2), (3), and (4) take the form, respectively:

$$\frac{\partial^2 \tau}{\partial u^2} + \frac{\partial}{\partial x}\left(\frac{\rho u \mu}{\tau}\right) - \frac{dp}{dx}\,\frac{\partial}{\partial u}\left(\frac{\mu}{\tau}\right) = 0, \tag{6}$$

$$\tau^2\left[\frac{\partial}{\partial u}\left(\frac{1}{\mathrm{Pr}}\,\frac{\partial h}{\partial u}\right) + 1\right] + \\ + (1-\mathrm{Pr})\left(\frac{1}{\mathrm{Pr}}\,\frac{\partial h}{\partial u}\right)\frac{\partial \tau}{\partial u}\,\tau - \rho u \mu\,\frac{\partial h}{\partial x} = 0, \tag{7}$$

$$\frac{\partial}{\partial u}\left[\frac{\partial}{\partial u}\left(\frac{C_1}{\mathrm{Pr}_m}\right)\right] + \\ + (1-\mathrm{Pr}_m)\,\frac{\partial}{\partial u}\left(\frac{C_1}{\mathrm{Pr}_m}\right)\frac{1}{\tau}\,\frac{\partial \tau}{\partial u} - \frac{\mu}{\tau^2}\,\rho u\,\frac{\partial C_1}{\partial x} = 0, \tag{8}$$

where τ is the shear stress. At the wall, in the presence of blowing [4],

$$\tau = \tau_w + (\rho v)_w\, u.$$

These equations are simplified appreciably if we assume

$$\frac{\partial h}{\partial x} = 0, \quad \frac{\partial C_1}{\partial x} = 0. \tag{9}$$

It has been shown [1, 4] that this assumption is admissible for the conditions examined, and results in h and C becoming functions of the single variable u. Moreover, since $\mu = \mu_1(T) = \mu_2(u)$ and $\tau \rightarrow \infty$ when $x \rightarrow 0$, Eq. (6) takes the form [4]

$$KK'' + 2\eta\rho_0\mu_0 = 0, \tag{10}$$

where $\eta = u/u_\infty$, $\rho_0 = \rho/\rho_\infty$, $\mu_0 = \mu/\mu_\infty$, $K = 2\sqrt{x/\rho_\infty\mu_\infty u_\infty^3}\,\tau$. (The primes denote differentiation with respect to η). It should be noted that K is a function of the blowing parameter. With (9) taken into account, the equations of energy (7) and diffusion (8) may be transformed to

$$\left(\frac{h'}{\mathrm{Pr}}\right)' + (1-\mathrm{Pr})\,\frac{k'}{k}\left(\frac{h'}{\mathrm{Pr}}\right) = -u_\infty^2, \tag{11}$$

$$\left(\frac{C_1'}{\mathrm{Pr}_m}\right) + (1-\mathrm{Pr}_m)\,\frac{k'}{k}\left(\frac{C_1'}{\mathrm{Pr}_m}\right) = 0 \tag{12}$$

with the boundary conditions

$$h = h_\infty,\ K = 0 \text{ when } \eta = 1; \tag{13}$$

$$K_{\mathrm{I}} = K_{\mathrm{II}},\ C_1 = 0,\ \eta_{\mathrm{I}} = \eta_{\mathrm{II}},\ h_{\mathrm{I}} = h_{\mathrm{II}}; \tag{14}$$

$$h_{\mathrm{I}}' - h_{\mathrm{II}}' = \frac{\mathrm{Pr}}{\mathrm{Pr}_m}\, QC_1' \text{ when } \eta = \eta_*; \tag{15}$$

$$h = h_w,\ K' = 2(\rho v)_w\sqrt{x/\rho_\infty\mu_\infty u_\infty^3},$$

$$(\rho v)_w = \text{const} = (\rho v)_w C_1(0) - C_1'(0)\,\tau_w/\mathrm{Pr}_m \text{ when } \eta = 0. \tag{16}$$

Equations (14) and (15), which are written on the basis of studies of laminar motion in a boundary layer with a discontinuity surface [5, 6], follow from the laws of conservation of momentum ($K_{\mathrm{I}} = K_{\mathrm{II}}$) and energy. Relation (15) has been introduced to determine the location of the reaction front in the boundary layer. The two successive conditions in (14) determine the uniqueness of the solution.

The expression for the mass flow of component i (j_i) passing through some surface parallel to the wall is

$$j_i = \rho v = C_i \rho v - \rho D_i\,\frac{\partial C_i}{\partial y} = C_i \rho v - \frac{\tau}{\mathrm{Pr}_m}\,\frac{\partial C_i}{\partial u}. \tag{17}$$

The boundary condition $(\rho v)_W = \mathrm{const}$ in (16) was obtained for an injected gas on the basis of (17). The solution of the nonlinear momentum equation (10) in regions I and II, taking into account (13)–(16), may be obtained by the method of numerical integration. If the case $\mu\rho = \mathrm{const}$ or $\mu_0\rho_0 = 1$ is to be examined, Eq. (10) is put in the form

$$KK'' + 2\eta = 0, \tag{18}$$

which is independent of the thermal and diffusion problems. This second-order ordinary differential equation is equivalent to the Blasius equation and may be obtained directly from it by simple transformations [7].

To compute the functions $k(\eta)$, we can use tables prepared for the Blasius problem [11]. An approximate expression for $k(\eta)$ without allowance for blowing has the form [7]

$$K(\eta) = K_w\sqrt{1-\eta^2},$$

where $K_w = c_f\sqrt{\mathrm{Re}_x} = 0.664$.

When $\eta \to 1$, we obtain from (18) the approximate equation $KK'' + 2 = 0$, which may be integrated in finite form [4].

We solve the energy equation (11) for regions II and I with the boundary conditions (13)–(15) and (15), (16), respectively, and the diffusion equation (12) in region I with boundary conditions (14)–(16).

For region I

$$\frac{h'}{\mathrm{Pr}} = \left\{-u_\infty^2 \int\limits_0^{\eta}\left[\exp(1-\mathrm{Pr})\,\frac{dK}{K}\right]d\eta + A\right\}\exp\left[-\int\limits_{K(0)}^{K(\eta)}(1-\mathrm{Pr})\,\frac{dK}{K}\right].$$

From repeated integration we obtain

$$h = h_w + A\cdot S(0,\ \eta) - u_\infty^2\,R(0,\ \eta), \tag{19}$$

where

$$S(0,\ \eta) = \int\limits_0^{\eta}\mathrm{Pr}\exp\left[-\int\limits_{K(0)}^{K(\eta)}(1-\mathrm{Pr})\,\frac{dK}{K}\right]d\eta, \tag{20}$$

$$R(0,\ \eta) = \int\limits_0^{\eta}\left\{\mathrm{Pr}\exp\left[-\int\limits_{K(0)}^{K(\eta)}(1-\mathrm{Pr})\,\frac{dK}{K}\right]\int\limits_0^{\eta}\times\right.$$

$$\left.\times\exp\left[\int\limits_{K(0)}^{K(\eta)}(1-\mathrm{Pr})\,\frac{dK}{K}\right]d\eta\right\}d\eta, \tag{21}$$

$$A = \frac{h'(0)}{\mathrm{Pr}(0)} = \frac{h_* - h_w + u_\infty^2\,R(0,\ \eta_*)}{S(0,\ \eta_*)} \tag{22}$$

is a constant of integration; we have $S(0,\ \eta_*)$ and $R(0,\ \eta_*)$ from (20) and (21) at a value of the limit of integration $\eta = \eta_*$.

Using (22), we put (19) in the form

$$h = h_w - u_\infty^2 R(0,\ \eta) + [h_* - h_w + u_\infty^2 R(0,\ \eta_*)]\,S(0,\ \eta)/S(0,\ \eta_*). \tag{23}$$

Similarly, we obtain the enthalpy distribution in region II:

$$h = h_* - u_\infty^2 R(\eta_*,\ \eta) + [h_* - h_w + u_\infty^2 R(0,\ \eta_*)]\,S(0,\ \eta)/S(0,\ \eta_*), \tag{24}$$

where

$$S(\eta_*,\ \eta) = \int\limits_{\eta_*}^{\eta}\mathrm{Pr}\exp\left[\int\limits_{K(\eta_*)}^{K(\eta)}(\mathrm{Pr}-1)\,\frac{dK}{K}\right]d\eta, \tag{25}$$

$$R(\eta_*,\ \eta) = \int\limits_{\eta_*}^{\eta} \left\{ \Pr \exp \left[\int\limits_{K(\eta_*)}^{K(\eta)} (\Pr - 1) \frac{dK}{K} \right] \times \int\limits_{\eta_*}^{\eta} \exp \left[\int\limits_{K(\eta_*)}^{K(\eta)} (1 - \Pr) \frac{dK}{K} \right] d\eta \right\} d\eta. \tag{26}$$

We find functions $S(\eta_*,\ 1)$ and $R(\eta_*,\ 1)$ from (25) and (26), taking the upper limit of integration as $\eta = 1$.

We obtain the concentration distribution in the boundary layer from the solution of (12) and (14)-(16):

$$C_1 = -u_\infty \xi_w S_m(\eta_*,\ \eta)\,[1 + u_\infty \xi_w S_m(0,\ \eta_*)]^{-1}, \tag{27}$$

where $\xi_w = (\rho v)_w / \tau_w$,

$$S_m(\eta_*,\ \eta) = \int\limits_{\eta_*}^{\eta} \Pr_m \exp \left[\int\limits_{K(\eta_*)}^{K(\eta)} (\Pr_m - 1) \frac{dK}{K} \right] d\eta;$$

$$S_m(0,\ \eta_*) = \int\limits_{0}^{\eta_*} \Pr_m \exp \left[\int\limits_{K(0)}^{K(\eta)} (\Pr_m - 1) \frac{dK}{K} \right] d\eta.$$

From (15), using (23) and (27), we obtain a relation for determining the position of the reaction front:

$$\frac{u_\infty^2 R(\eta_*,\ 1) + h_\infty - h_*}{S(\eta_*,\ 1)} \exp \left[\int\limits_{K(0)}^{K(\eta_*)} (1 - \Pr) \frac{dK}{K} \right] -$$

$$- \frac{u_\infty^2 R(0,\ \eta_*) + h_* - h_w}{S(0,\ \eta_*)} + u_\infty^2 \int\limits_{0}^{\eta_*} \exp \left[\int\limits_{K(0)}^{K(\eta)} (1 - \Pr) \frac{dK}{K} \right] d\eta = \tag{28}$$

$$= Q u_\infty \xi_w \left\{ \exp \left[\int\limits_{K(0)}^{K(\eta_*)} (\Pr_m - \Pr) \frac{dK}{K} \right] \right\} [1 + u_\infty \xi_w S_m(0,\ \eta_*)]^{-1}.$$

If Pr and $\Pr_m$ are constant, functions $S(\eta_1,\ \eta_2)$, $S_m(\eta_1,\ \eta_2)$ and $R(\eta_1,\ \eta_2)$, where η_1, η_2 are the limits of integration, may be written as

$$S(\eta_1,\ \eta_2) = \Pr I_m(\eta_1,\ \eta_2) = \Pr \int\limits_{\eta_1}^{\eta_2} [k(\eta_2)/k(\eta_1)]^{\Pr - 1}\, d\eta,$$

$$S_m(\eta_1,\ \eta_2) = \Pr_m I_m(\eta_1,\ \eta_2) = \Pr_m \int\limits_{\eta_1}^{\eta_2} [k(\eta_2)/k(\eta_1)]^{\Pr_m - 1}\, d\eta,$$

$$R(\eta_1,\ \eta_2) = \Pr J(\eta_1,\ \eta_2) = \Pr \int\limits_{\eta_1}^{\eta_2} \left\{ [k(\eta_2)/k(\eta_1)]^{\Pr - 1} \int\limits_{\eta_1}^{\eta_2} [k(\eta_2)/k(\eta_1)]^{1 - \Pr} d\eta \right\} d\eta.$$

Functions I and J have been tabulated [4, 7] for various fixed values of the Prandtl number and for shear stress distributions according to the Blasius law. In the case when Pr = const and $\Pr_m$ = const, (23) and (24) transform into the Crocco formulas [4]

$$h = h_w - u_\infty^2 \Pr J(0,\ \eta) + [h_* - h_w + u_\infty^2 \Pr J(0,\ \eta_*)]\, \frac{I(0,\ \eta)}{I(0,\ \eta_*)}, \tag{29}$$

$$h = h_* - u_\infty^2 \Pr J(\eta_*,\ \eta) + [h_\infty - h_* + u_\infty^2 \Pr J(\eta_*,\ 1)]\, \frac{I(\eta_*,\ \eta)}{I(\eta_*,\ 1)}, \tag{30}$$

and the concentration distribution is given by the relation

$$C_1 = -I_m(\eta_*,\ \eta) \left[\frac{1}{\Pr_m u_\infty \xi_w} + I_m(0,\ \eta_*) \right]^{-1}, \tag{31}$$

while condition (28) takes the form

$$u_\infty^2 I(0,\ \eta_*) - \frac{u_\infty^2 \Pr J(0,\ \eta_*) + h_* - h_w}{\Pr I(0,\ \eta_*)} + \frac{u_\infty^2 \Pr J(\eta_*,\ 1) + h_\infty - h_*}{\Pr I(\eta_*,\ 1)} \times$$
$$\times \left[\frac{k(\eta_*)}{k(0)}\right]^{1-\Pr} = Q\xi_w \left[\frac{k(\eta_*)}{k(0)}\right]^{\Pr_m - \Pr} [1 + \xi_w \Pr_m u_\infty I(0,\ \eta_*)]. \tag{32}$$

Considerable mathematical simplification is possible when $\Pr = \Pr_m = 1$ or $\nu = a = D$. From (29)–(31) we obtain, respectively,

$$\frac{H - H_w}{H_* - H_w} = \frac{\eta}{\eta_*},\quad \frac{H - H_*}{H_\infty - H_*} = \frac{\eta - \eta_*}{1 - \eta_*},\quad C_1 = \frac{u_\infty \xi_w (\eta_* - \eta)}{1 + u_\infty \xi_w \eta_*},$$

i.e., the distribution of total enthalpy $H = c_p T + u^2/2$ and concentration, determined by integration in closed form, depends linearly on the flow velocity, which agrees with the data of [1, 5]. Condition (31) for determining **the** location of the reaction front is transformed to an algebraic equation of third degree in η_*:

$$\eta_*^3 u_\infty^2 (Q\xi_w^2 - 1) + \eta^2 u_\infty \left(\frac{u_\infty}{2} - Qu_\infty \xi_w^2 + Q\xi_w\right) + \eta_* \left(\frac{u_\infty^2}{2} - h_w + h_\infty - Q\xi_w u_\infty\right) + (h_w - h_*) = 0, \tag{33}$$

where the root η_* is so chosen that $0 < \eta_* < 1$.

The solution obtained allows us to calculate the heat flux $q_W = (H_* - H_W)\,\alpha/C_p$ from the gas to the plate surface.

The heat transfer coefficient is

$$\alpha = \left(\lambda \frac{\partial H}{\partial y}\right)_w (H_* - H_w) = \lambda \left(\frac{\partial H}{\partial \eta}\right)_w \left(\mu \frac{\partial u}{\partial y}\right)_w \Big/ \mu u_\infty (H_* - H_w). \tag{34}$$

From (22) and (34), taking account of the relation for $\tau_w = \frac{1}{2} K(0) \sqrt{\rho_\infty \mu_\infty u_\infty^3 / x}$ and for the Reynolds number $\mathrm{Re}_x = \rho_\infty u_\infty x/\mu_\infty$, we have

$$\alpha = \frac{c_p \rho_\infty u_\infty}{H_* - H_w} \frac{K(0)}{S(0,\ \eta_*) 2\sqrt{\mathrm{Re}_x}} \left[h_* + 2R(0,\ \eta_*) \frac{u_\infty^2}{2} - h_w\right]. \tag{35}$$

Having determined the Stanton number St_x and the recovery enthalpy h_r according to [4], i.e.,

$$\mathrm{St}_x = \frac{K(0)}{S(0,\ \eta_*) 2\sqrt{\mathrm{Re}_x}} = \frac{1}{S(0,\ \eta_*)} \frac{c_f}{2}, \tag{36}$$

$$h_r = h_* + 2R(0,\ \eta_*) \frac{u_\infty^2}{2}, \tag{37}$$

we transform (35) to

$$\alpha = c_p \rho_\infty u_\infty \mathrm{St}\,(h_r - h_w)/(H_* - H_w), \tag{38}$$

where the values $S(0,\ \eta_*) = s$ and $2R(0,\ \eta_*) = r$ may be regarded, respectively, as the coefficients of the Reynolds analogy and recovery enthalpy.

When a liquid filters through the porous wall and vaporizes at the surface in such a way that no liquid film of coolant is formed, the corresponding heat flux will be $q_{W_2} = -(\rho v)_W L$.

Having determined the heat transfer at the porous wall with material injected according to the relation $q_{W_3} = \pm(\rho v)_W l$, we can calculate the total heat flux $q_W = q_{W_1} + q_{W_2} + q_{W_3}$ or

$$q_w = \frac{\alpha}{c_p} (H_* - H_w) - (\rho v)_w (L \pm l)^*.$$

We shall now consider a turbulent boundary layer in relation to the conditions of the present problem. Taking account of (10), we can write the equations of momentum, energy, and diffusion in the Crocco variables for the averaged

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[*] Heat transfer due to radiation is not taken into account.

steady state, respectively, in the form [4]

$$\frac{\partial}{\partial x}\left[\frac{\rho \bar{u}(\mu+\varepsilon_\mu)}{\tau}\right]+\frac{\partial^2 \tau}{\partial \bar{u}^2}=0, \tag{39}$$

$$\left(\frac{\bar{h}'}{\mathrm{Pr}^c}\right)'+(1-\mathrm{Pr}^c)\frac{\tau'}{\tau}\left(\frac{\bar{h}'}{\mathrm{Pr}^c}\right)=-\bar{u}_\infty^2, \tag{40}$$

$$\left(\frac{\bar{C}_1'}{\mathrm{Pr}_m^c}\right)'+(1-\mathrm{Pr}_m^c)\frac{\tau'}{\tau}\left(\frac{\bar{C}_1'}{\mathrm{Pr}_m^c}\right)=0, \tag{41}$$

where

$$\mathrm{Pr}^c=\frac{c_p(\bar{\mu}+\varepsilon_\mu)}{\bar{\lambda}+\varepsilon_\lambda},\quad \mathrm{Pr}_m^c=\frac{\bar{\mu}-\varepsilon_\mu}{\bar{\rho}(\bar{D}+\varepsilon_D)}, \tag{42}$$

$$-\overline{(\rho v)'u'}=\varepsilon_\mu\frac{\partial \bar{u}}{\partial y},\quad -\overline{(\rho v)'h'}=\varepsilon_\lambda\frac{\partial T}{\partial y},\quad -\overline{(\rho v)'C_i'}=\bar{\rho}\,\varepsilon_D\frac{\partial \bar{C}_i}{\partial y}. \tag{43}$$

In (39)-(43) the bar denotes averaging, and the primes in (42) and (43) the fluctuating component.

In the case examined the momentum equation (39) does not give a solution. However, integration of the equations of energy (40) and diffusion (41) with boundary conditions (13)-(16), transformed for turbulent flow, may be carried out in the same way as for the laminar boundary layer. It is evident that integration results in analytical expressions for $\bar{h}_1\bar{C}_1$ and $\bar{\eta}_*$ that are completely analogous to those obtained for the laminar boundary layer (23)-(28), but written for averaged values and with Pr^c and Pr_m^c instead of Pr and Pr_m. Therefore, relations (36)-(38), which determine

$$\mathrm{St}=\frac{1}{s}\frac{c_f}{2},\quad h_r=h_*+r\frac{u_\infty^2}{2},\quad \alpha=c_p\rho_\infty u_\infty \mathrm{St}\,(h_r-h_w)(H_*-H_w),$$

will also be valid for the turbulent boundary layer. Thus, to calculate the heat transfer coefficient α (or q_w), it is necessary to determine the coefficients r, s, and c_f. A method of calculating these coefficients for the case of a nonreacting turbulent boundary layer was given in [4].

Notation

x—coordinate along the plate; y—coordinate normal to the plate; u, v—flow velocity components along x and y, respectively; ρ, T, λ, a, μ—respectively, density of mixture, absolute temperature, thermal conductivity, thermal diffusivity, and viscosity; c_p—specific heat at constant pressure; D—diffusion coefficient; c_p—specific heat at constant pressure; D—diffusion coefficient; R—universal gas constant; $\mathrm{Pr} = \mu c_p/\lambda$—thermal, and $\mathrm{Pr}_m = \mu/\rho D$—diffusion Prandtl number; ε_μ, ε_λ, ε_D—turbulent viscosity, thermal conductivity, and diffusion, respectively (see Eq. (43)); Pr^c, Pr_m^c—compound Prandtl numbers, thermal and diffusion, respectively (see Eq. (42)); Q—heat of reaction; L—heat of vaporization; l—heat emitted or received by the wall; c_f—local coefficient of surface friction $c_f = 2\tau_w/\rho_\infty u_\infty^2$; $h_w = H_w = c_pT_w$; $H_\infty = c_pT_\infty + u_\infty^2/2$; $H_* = c_pT_* + u_*^2/2$. Subscripts: ∞—conditions at outer edge of boundary layer; w—conditions on surface; x—value at a given coordinate; *—conditions in reaction zone; I—values between reaction front and plate; II—above reaction front in the boundary layer.

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